# 2 Polynomials

## Fastrack Revision

- **Algebraic Expression:** Any expression that contains constants and variables, connected by some or all of the operations $+$, $-$, $\times$ and $\div$, is known as an algebraic expression. For example, $x^2+1$, $6x^2-5y^2+2xy$ etc.
- **Polynomials:** An algebraic expression in which the variables involved have only non-negative integral powers. For example, $7x+y+5$, $a+b$, etc.
- **Polynomial in One Variable:** An algebraic expression which consists of only one type of variable in the entire expression. For example, $2x^2+5x-7$, $3y^3+12y^2+7y-9$, etc.
- **General Expression of Polynomial:** A polynomial in one variable $x$ of degree $n$ can be expressed as

$$a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$$

where, $a_n \neq 0$, $a_0$ is constant term and $a_1, a_2, \dots\dots, a_n$ are called coefficients of $x, x^2, x^3, \dots x^n$ respectively.

- **Terms and their Coefficients:** If $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1 x + a_0$ is a polynomial in variable $x$, then $a_n x^n$, $a_{n-1}x^{n-1}$, $a_{n-2}x^{n-2}$, ...., $a_1 x$ and $a_0$ are known as the terms of polynomial $f(x)$ and $a_n, a_{n-1}, a_{n-2}, \dots, a_1$ and $a_0$ respectively are known as their coefficients.
- **Degree of a Polynomial:** Highest power of a variable in the polynomial.

  For example, in case of $6x^4+5x^3+3$, the highest power of $x$ is 4, so the degree of polynomial is 4.
- **Types of Polynomials:**
  1. **Linear Polynomial:** Polynomial of degree 1. In general, $ax+b$, $a \neq 0$ is a linear polynomial.
  2. **Quadratic Polynomial:** Polynomial of degree 2. In general, $ax^2+bx+c$, $a \neq 0$ is a quadratic polynomial.
  3. **Cubic Polynomial:** Polynomial of degree 3. In general, $ax^3+bx^2+cx+d$, $a \neq 0$ is a cubic polynomial.
  4. **Biquadratic Polynomial:** Polynomial of degree 4. In general, $ax^4+bx^3+cx^2+dx+e$, $a \neq 0$ is a biquadratic polynomial.
  5. **Constant Polynomial:** Polynomial of degree 0, consisting of a non-zero constant. For example, 12, –7, 9/14, etc.
  6. **Zero Polynomial:** A polynomial consisting of one term, namely zero.
- **Classification of Polynomials:**
  1. **Monomial:** Polynomial with only one term. For example, $5x$, $\frac{3}{8}y$, etc.
  2. **Binomial:** Polynomial having two terms. For example, $8x+5y-7y^2+8y$ etc.
  3. **Trinomial:** Polynomial having three terms. For example, $3y^2+4y+\frac{19}{7}$ etc.

### Knowledge BOOSTER

1. *Linear polynomial can be monomial or binomial.*
2. *Quadratic polynomial can be monomial, binomial or trinomial.*
3. *The degree of a zero polynomial is not defined.*

- **Zeroes of a Polynomial:** Let $p(x)$ be a polynomial in one variable and '$\alpha$' be a real number such that the value of polynomial at $x=\alpha$ is zero, *i.e.*, $p(\alpha)=0$, then '$\alpha$' is said to be a zero of a polynomial $p(x)$.

### Knowledge BOOSTER

1. *A non-zero constant polynomial has no zero.*
2. *0 may or may not be the zero(s) of a given polynomial.*
3. *A polynomial of nth degree can have maximum n zeroes.*

- **Division Algorithm in Polynomials:** Suppose $p(x)$ and $g(x)$ are two polynomials such that degree $p(x) \geq$ degree $g(x)$. When we divide $p(x)$ by $g(x)$, then we get the result in the form of

$$p(x) = g(x).\ q(x) + r(x)$$

where $q(x)$ = quotient

and $r(x)$ = remainder

- **Remainder Theorem:** Let $p(x)$ be a polynomial having degree 1 or more than 1 and let $\alpha$ be any real number. If $p(x)$ is divided by $(x-\alpha)$, then remainder is $p(\alpha)$.
- **Factor Theorem:** Suppose $p(x)$ be a polynomial of degree 1 or more than 1 and $\alpha$ be any real number.
  (i) If $p(\alpha)=0$, then $(x-\alpha)$ is a factor of $p(x)$.
  (ii) If $(x-\alpha)$ is a factor of $p(x)$, then $p(\alpha)=0$.

- **Factorisation of Quadratic Polynomial:** Quadratic polynomial can be factorised either by splitting middle term or by using factor theorem.
  1. **Polynomial of the form $x^2 + bx + c$:** We find integers $p$ and $q$ such that $p + q = b$ and $pq = c$.

     Then, $x^2 + bx + c = x^2 + (p + q)x + pq$

     $= x^2 + px + qx + pq$

     $= x(x + p) + q(x + p)$

     $= (x + p)(x + q)$
  2. **Polynomial of the form $ax^2 + bx + c$:** We find integers $p$ and $q$ such that $p + q = b$ and $pq = ac$.

     Then, $ax^2 + bx + c = ax^2 + (p + q)x + \frac{pq}{a}$

     $= \frac{a^2x^2 + apx + aqx + pq}{a}$

     $= \frac{ax(ax + p) + q(ax + p)}{a}$

     $= \frac{1}{a}(ax + p)(ax + q)$
- **Factorisation of Cubic Polynomial:** To factorise a cubic polynomial $p(x)$, we
  (i) find $x = a$, where $p(a) = 0$
  (ii) then $(x - a)$ is a factor of $p(x)$.
  (iii) now, divide $p(x)$ by $(x - a)$ *i.e.*, $p(x)/(x - a)$.
  (iv) and then we factorise the quotient polynomial by splitting the middle term.
- **Algebraic Identities:** Algebraic equations that are true for all values of variables occurring in it.

  Some useful algebraic identities are:
  (i) $(x + y)^2 = x^2 + 2xy + y^2$
  (ii) $(x - y)^2 = x^2 - 2xy + y^2$
  (iii) $x^2 - y^2 = (x + y)(x - y)$
  (iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$
  (v) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
  (vi) $(x + y)^3 = x^3 + y^3 + 3xy(x + y) = x^3 + y^3 + 3x^2y + 3xy^2$
  (vii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y) = x^3 - y^3 - 3x^2y + 3xy^2$
  (viii) $x^3 + y^3 = (x + y)(x^2 - xy + y^2) = (x + y)^3 - 3xy(x + y)$
  (ix) $x^3 - y^3 = (x - y)(x^2 + xy + y^2) = (x - y)^3 + 3xy(x - y)$
  (x) $(x^3 + y^3 + z^3 - 3xyz) = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

  If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

# Practice Exercise

## Multiple Choice Questions

**Q 1.** Which one is not a polynomial?

a. $4x^2 + 2x - 1$ b. $y + \frac{3}{y}$
c. $x^3 - 1$ d. $y^2 + 5y + 1$

**Q 2.** Which of the following algebraic expression is a polynomial in variable $x$?

a. $x^2 + \frac{2}{x^2}$ b. $\sqrt{x} + 1$
c. $x^3 + \frac{3x^{3/2}}{\sqrt{x}}$ d. $x^{-2} + x^{-1} + 5$

**Q 3.** The polynomial $px^2 + qx + rx^4 + 5$ is of type:

a. linear b. quadratic
c. cubic d. biquadratic

**Q 4.** The polynomial of type $p(x) = ax^2 + bx + c, a = 0$ is:

a. linear b. quadratic
c. cubic d. biquadratic

**Q 5.** Classification of the polynomial $3x^4 + 2x$ is:

a. monomial b. binomial
c. trinomial d. All of these

**Q 6.** The zero of the polynomial $p(x) = 2x + 5$ is:

a. 2 b. 5
c. $\frac{2}{5}$ d. $-\frac{5}{2}$

**Q 7.** The zeroes of the polynomial $p(x) = 3x^2 - 1$ are:

a. $\frac{1}{3}$ and 3 b. $\frac{1}{\sqrt{3}}$ and $\sqrt{3}$
c. $\frac{-1}{\sqrt{3}}$ and $\sqrt{3}$ d. $\frac{1}{\sqrt{3}}$ and $\frac{-1}{\sqrt{3}}$

**Q 8.** If $p(y) = 4 + 3y - y^2 + 5y^3$, then value of $p(-1)$ is:

a. $-5$ b. $-9$
c. 0 d. $-6$

**Q 9.** The number of zeroes in $x^2 + 4x + 4$ is:

a. 1 b. 2
c. 3 d. None of these

**Q 10.** If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to:

a. 3 b. $2x$ c. 0 d. 6

**Q 11.** The degree of polynomial $p(x) = x + \sqrt{x^2} + 1$ is:

a. 0 b. 2 c. 1 d. 3

**Q 12.** The remainder when $x^3 - 2x^2 + 3x + 2$ is divided by $x - 1$, is:

a. 4 b. $-4$
c. 3 d. $-3$

**Q 13.** If $x^{21} - 15$ is divided by $(x + 1)$, then remainder is:

a. $-14$ b. $-16$
c. $-15$ d. $17$

**Q 14.** If $(x - 2)$ is a factor of $x^3 - 2x^2 + p$, then the value of $p$ is:

a. $-1$ b. $1$
c. $-2$ d. $0$

**Q 15.** If $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, then the value of $b$ is:

a. $0$ b. $\frac{1}{\sqrt{2}}$
c. $\frac{1}{4}$ d. $\frac{1}{2}$

**Q 16.** The value of $5.63 \times 5.63 + 11.26 \times 2.37 + 2.37 \times 2.37$ is:

a. 237 b. 126
c. 56 d. 64

**Q 17.** If $x + y = 3$, $x^2 + y^2 = 5$, then $xy$ is:

a. 1 b. 3 c. 2 d. 5

**Q 18.** The coefficient of $x$ in the expansion of $(x + 3)^3$ is:

a. 1 b. 9 c. 18 d. 27

**Q 19.** The value of $\frac{(361)^3 + (139)^3}{(361)^2 - 361 \times 139 + (139)^2}$ is:

a. 300 b. 500
c. 400 d. 600

**Q 20.** Factors of $x^4 - x^2 - 12$ are:

a. $(x + 2), (x - 2), (x^2 + 3)$
b. $(x + 3), (x - 3), (x^2 + 2)$
c. $(x^2 + 2), (x^2 - 6)$
d. $(x + 2), (x - 2), (x^2 - 3)$

**Q 21.** Factorisation of $x^2 + 3\sqrt{2}x + 4$ is:

a. $(x + 2\sqrt{2})(x + \sqrt{2})$ b. $(x + 2\sqrt{2})(x - \sqrt{2})$
c. $(x - 2\sqrt{2})(x + \sqrt{2})$ d. $(x - 2\sqrt{2})(x - \sqrt{2})$

**Q 22.** One of the dimensions of the cuboid whose volume is $16x^2 - 26x + 10$, is:

a. $(x - 1)$ b. 2
c. $(8x - 5)$ d. All of these

**Q 23.** Factors of $(a + b)^3 - (a - b)^3$ are:

a. $ab$, $3a^2 + b^2$ b. $(3a^2 + b^2)$, $2a$
c. $2b$, $(3a^2 + b^2)$ d. $(3a^2 + b^2)$, $2ab$

**Q 24.** If $3x - 2y + z = 0$, then the value of $27x^3 - 8y^3 + z^3$ is:

a. $-6xyz$ b. $-18xyz$
c. $18xyz$ d. $6xyz$

## Assertion & Reason Type Questions

**Directions (Q. Nos. 25-29):** *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

**Q 25.** Assertion (A): The expression $3x^4 - 4x^{3/2} + x^2 - 2$ is not a polynomial because the term $-4x^{3/2}$ contains a rational power of $x$.

Reason (R): The highest exponent in various terms of an algebraic expression in one variable is called its degree.

**Q 26.** Assertion (A): The degree of the polynomial $(x - 2)(x - 3)(x + 4)$ is 4.

Reason (R): The number of zeroes of a polynomial is the degree of that polynomial.

**Q 27.** Assertion (A): If $p(x) = x^2 - 4x + 3$, then 3 and 1 are the zeroes of the polynomial $p(x)$.

Reason (R): Number of zeroes of a polynomial cannot exceed its degree.

**Q 28.** Assertion (A): The remainder when $p(x) = x^3 - 2x^2 + 3x$ is divided by $(2x - 1)$ is $\frac{9}{8}$.

Reason (R): If a polynomial $p(x)$ is divided by $ax - b$, the remainder is the value of $p(x)$ at $x = \frac{b}{a}$.

**Q 29.** Assertion (A): Factorisation of the polynomial $\sqrt{3}x^2 + 11x + 6\sqrt{3}$ is $(\sqrt{3}x + 2)(x + \sqrt{3})$.

Reason (R): Factorisation of the polynomial $35y^2 + 13y - 12$ is $(7y - 3)(5y + 4)$.

## Fill in the Blanks Type Questions

**Q 30.** Every real number is a .................. of the zero polynomial.

**Q 31.** A polynomial of degree 3 in $x$ has at most .................. terms.

**Q 32.** The quotient of $8x^3 - 5x^2 + 2x$, when divided by $2x$ is ................. .

**Q 33.** If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of $k$ is .................. .

**Q 34.** If $\frac{x}{y} + \frac{y}{x} = -1$, where $x, y \neq 0$, then the value of $(x^3 - y^3)$ is ..................

## True/False Type Questions

**Q 35.** A polynomial of $n$th degree can have maximum $n$ zeroes.

**Q 36.** The integral zeroes of the polynomial $x^3 + 3x^2 - x - 3$ are $-1$, 1 and $-3$.

**Q 37.** The zeroes of the polynomial $p(x) = 2x^2 + 7x - 4$ are $-4$ and $\frac{1}{2}$.

**Q 38.** $(x + 3)$ is a factor of $x^3 + 2x^2 - 4x + 3$.

**Q 39.** When $p(x)$ is divided by $(x - 2)$, then remainder is $p(2)$.

## Solutions

**1.** (b) $y + \frac{3}{y}$

**2.** (c) $x^3 + \frac{3x^{3/2}}{\sqrt{x}}$ can be written as $x^3 + 3x$, which is a polynomial in variable $x$.

**3.** (d) biquadratic

**4.** (a) Here, it is given that $a = 0$

$\Rightarrow \quad p(x) = ax^2 + bx + c$

$= bx + c,$

which is a linear polynomial.

**5.** (b) binomial

**6.** (d) Consider $\quad p(x) = 0$

$\Rightarrow \quad 2x + 5 = 0$

$\Rightarrow \quad x = -\frac{5}{2}$

**7.** (d) Given,

$p(x) = 3x^2 - 1,$

Now, $P\left(\frac{1}{\sqrt{3}}\right) = 3\left(\frac{1}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{1}{3} - 1 = 1 - 1 = 0$

$P\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{1}{3} - 1 = 1 - 1 = 0$

$\Rightarrow \frac{1}{\sqrt{3}}$ and $\frac{-1}{\sqrt{3}}$ are the zeroes of the polynomial.

**8.** (a) $p(-1) = 4 + 3(-1) - (-1)^2 + 5(-1)^3$

$= 4 - 3 - 1 - 5 = 4 - 9 = -5$

**9.** (b) Given,

$p(x) = x^2 + 4x + 4$

$= x^2 + 2 \times 2 \times x + 2^2$

$= (x + 2)^2$

Consider,

$p(x) = 0$

$\Rightarrow \quad (x + 2)^2 = 0 \quad \Rightarrow \quad x = -2, -2$

Hence, it has two zeroes.

**10.** (d) Given,

$p(x) = x + 3$

$p(-x) = -x + 3$

$\therefore \quad p(x) + p(-x) = x + 3 + (-x + 3)$

$= x + 3 - x + 3 = 6$

**11.** (c) $p(x) = x + \sqrt{x^2} + 1$

$= x + (x^2)^{\frac{1}{2}} + 1$

$= x + x + 1$

$= 2x + 1$

Hence, the degree of a polynomial is one.

**12.** (a) Let $p(x) = x^3 - 2x^2 + 3x + 2$

> **TiP**
>
> *If $p(x)$ is divided by $x - a$, then remainder is $p(a)$.*

When $p(x)$ is divided by $(x - 1)$, then remainder is

$p(1) = (1)^3 - 2(1)^2 + 3(1) + 2$

$= 1 - 2 + 3 + 2 = 4$

**13.** (b) Given, $x^{21} - 15$ is divided by $(x + 1)$, then remainder is $(-1)^{21} - 15 = -1 - 15 = -16$

**14.** (d)

> **TiP**
>
> *If $(x - \alpha)$ is a factor of $p(x) = ax^2 + bx + c$, then $p(\alpha) = 0$.*

Let $g(x) = x^3 - 2x^2 + p$

Since, $(x - 2)$ is a factor of $g(x)$. Therefore $g(2) = 0$

$\Rightarrow \quad (2)^3 - 2(2)^2 + p = 0$

$\Rightarrow \quad 8 - 8 + p = 0$

$\Rightarrow \quad p = 0$

**15.** (c) $\left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right) = (7x)^2 - \left(\frac{1}{2}\right)^2$

$[\because (a + b)(a - b) = a^2 - b^2]$

$= 49x^2 - \frac{1}{4}$ ...(1)

On comparing eq. (1) with $49x^2 - b$, we get

$b = \frac{1}{4}$

**16.** (d) $5.63 \times 5.63 + 11.26 \times 2.37 + 2.37 \times 2.37$

$= (5.63)^2 + 2 \times 5.63 \times 2.37 + (2.37)^2$

$= (5.63 + 2.37)^2 \quad [\because a^2 + 2ab + b^2 = (a + b)^2]$

$= (8)^2 = 64$

17. (c) We know that, $(x+y)^2 = x^2 + y^2 + 2xy$

$$\Rightarrow \quad xy = \frac{(x+y)^2 - (x^2+y^2)}{2}$$

$$= \frac{(3)^2 - 5}{2} = \frac{9-5}{2} = 2$$

18. (d) $(x+3)^3 = x^3 + 9x^2 + 27x + 27$

Coefficient of $x = 27$

19. (b) $\dfrac{(361)^3 + (139)^3}{(361)^2 - 361 \times 139 + (139)^2}$

$[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$$= \frac{(361+139)[(361)^2 - 361\times 139 + (139)^2]}{[(361)^2 - 361\times 139 + (139)^2]}$$

$= (361 + 139) = 500$

20. (a) $x^4 - x^2 - 12 = x^4 - 4x^2 + 3x^2 - 12$

$= x^2(x^2 - 4) + 3(x^2 - 4) = (x^2 - 4)(x^2 + 3)$

$= (x+2)(x-2)(x^2+3).$

21. (a) $x^2 + 3\sqrt{2}x + 4 = x^2 + 2\sqrt{2}x + \sqrt{2}x + 4$

$= x(x + 2\sqrt{2}) + \sqrt{2}(x + 2\sqrt{2})$

$= (x + \sqrt{2})(x + 2\sqrt{2})$

22. (d) $16x^2 - 26x + 10 = 2(8x^2 - 13x + 5)$

$= 2(8x^2 - 8x - 5x + 5)$

$= 2[8x(x-1) - 5(x-1)]$

$= 2(8x - 5)(x - 1)$

23. (c) $(a+b)^3 - (a-b)^3$

$= [(a+b) - (a-b)][(a+b)^2 + (a+b)(a-b) + (a-b)^2]$

$= 2b[a^2 + b^2 + 2ab + a^2 - b^2 + a^2 + b^2 - 2ab]$

$= 2b(3a^2 + b^2)$

24. (b)

**TR!CK**

*If* $x + y + z = 0$, *then*
$x^3 + y^3 + z^3 = 3xyz$

Given, $3x - 2y + z = 0$

$\therefore \quad 27x^3 - 8y^3 + z^3 = (3x)^3 + (-2y)^3 + (z)^3$

$= 3(3x)(-2y)(z) = -18xyz$

25. (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

26. (d) **Assertion (A):** $p(x) = (x-2)(x-3)(x+4)$

$= (x-2)[x^2 + 4x - 3x - 12]$

$= (x-2)(x^2 + x - 12)$

$= x^3 + x^2 - 12x - 2x^2 - 2x + 24$

$p(x) = x^3 - x^2 - 14x + 24$

So, degree of $p(x) = 3$.

Hence, Assertion (A) is false, but Reason (R) is true.

27. (b) **Assertion (A):** Given, $p(x) = x^2 - 4x + 3$

$\Rightarrow \quad p(x) = x^2 - (3+1)x + 3$

$= x^2 - 3x - x + 3$

$= x(x-3) - 1(x-3)$

$= (x-1)(x-3)$

For finding the zeroes, put $p(x) = 0$

$\therefore (x-1)(x-3) = 0 \quad \Rightarrow \quad x = 1, 3$

So, Assertion (A) is true.

**Reason (R):** It is true to say that the number of zeroes of a polynomial cannot exceed its degree.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

28. (a) **Assertion (A):** When $p(x) = x^3 - 2x^2 + 3x$ is divided by $(2x - 1)$, then remainder is

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)$$

$$= \frac{1}{8} - \frac{1}{2} + \frac{3}{2}$$

$$= \frac{1}{8} + \frac{2}{2} = \frac{1}{8} + 1$$

$$= \frac{9}{8}$$

So, Assertion (A) is true.

**Reason (R):** It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

29. (d) **Assertion (A):**

$(\sqrt{3}x^2 + 11x + 6\sqrt{3}) = \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3}$

$= \sqrt{3}x(x + 3\sqrt{3}) + 2(x + 3\sqrt{3})$

$= (x + 3\sqrt{3})(\sqrt{3}x + 2).$

So, Assertion (A) is false.

**Reason (R):**

$(35y^2 + 13y - 12) = 35y^2 + 28y - 15y - 12$

$= 7y(5y+4) - 3(5y+4)$

$= (5y+4)(7y-3)$

So, Reason (R) is true.

30. zero

31. 4

32. Required quotient $= \dfrac{8x^3 - 5x^2 + 2x}{2x}$

$= 4x^2 - \dfrac{5}{2}x + 1$

33. Let $p(x) = x^2 + 3x + k$

Given, $p(2) = 0$

$\Rightarrow (2)^2 + 3(2) + k = 0$

$\Rightarrow 10 + k = 0$

$\Rightarrow k = -10$

34. $\frac{x}{y} + \frac{y}{x} = -1$

$\Rightarrow x^2 + y^2 + xy = 0$

Multiply both sides by $(x - y)$

$\Rightarrow (x - y)(x^2 + y^2 + xy) = 0$

$\Rightarrow x^3 - y^3 = 0$

35. True

36. True

Let $p(x) = x^3 + 3x^2 - x - 3$

$p(-1) = (-1)^3 + 3(-1)^2 - (-1) - 3$

$= -1 + 3 + 1 - 3$

$= 0$

$p(1) = (1)^3 + 3(1)^2 - (1) - 3$

$= 1 + 3 - 1 - 3$

$= 0$

$p(-3) = (-3)^3 + 3(-3)^2 - (-3) - 3$

$= -27 + 27 + 3 - 3 = 0$

37. True

$p(x) = 2x^2 + 7x - 4$

$p(x) = 2x^2 + 8x - x - 4$

$= 2x(x + 4) - 1(x + 4)$

$= (x + 4)(2x - 1).$

For zeroes, $p(x) = 0$

$\Rightarrow (x + 4)(2x - 1) = 0$

$\Rightarrow x = -4$

and $x = 1/2$

38. False

Let $p(x) = x^3 + 2x^2 - 4x + 3$. Then

$p(-3) = (-3)^3 + 2(-3)^2 - 4(-3) + 3$

$= -27 + 18 + 12 + 3 = 6$

Hence, $(x + 3)$ is not a factor of $p(x)$.

39. True

## Case Study Based Questions

### Case Study 1

A reputed school of Meerut decided to conduct different types of Tours for the students to educate them. So in class X, $\frac{1}{6}$th times the square of the total number of students planned to visit historical monuments, $\frac{5}{6}$th times the total number of students planned to visit old age homes while 10 students decided to teach poor children.


*On the basis of the above information, solve the following questions:*

**Q 1. Using above information, express the total number of students as a polynomial in terms of $x$:**

a. $\frac{x^2}{6} + \frac{5}{6}x + 10$  b. $\frac{x^2}{4} + \frac{7}{4}x + 10$

c. $\frac{7x^2}{12} + \frac{1}{12}x + 10$  d. $\frac{x^2}{4} + \frac{7}{4}x + 15$

**Q 2. Write the coefficient of $x$ in polynomial.**

a. $\frac{9}{13}$  b. $\frac{5}{6}$  c. $\frac{11}{12}$  d. $\frac{13}{12}$

**Q 3. Write the coefficient of $x^2$ in polynomial.**

a. $\frac{1}{13}$  b. $\frac{1}{10}$  c. $\frac{1}{6}$  d. 15

**Q 4. Value of $p(x)$ at $x = 2$ is:**

a. $\frac{37}{3}$  b. $\frac{11}{2}$  c. $\frac{22}{3}$  d. $\frac{14}{3}$

**Q 5. When $p(x)$ is divided by $x$, then quotient is:**

a. $\frac{x}{6} + \frac{5}{6} - \frac{10}{x}$  b. $\frac{x}{6} + \frac{5}{6} + \frac{10}{x}$

c. $\frac{x^3}{6} + \frac{5}{6}x^2 + 10x$  d. $\frac{x^3}{6} + \frac{5}{6}x + 10$

## Solutions

1. (a) Let the total number of students be $x$ then $\frac{1}{6}$th times the square of total students $= \frac{x^2}{6}$

and $\frac{5}{6}$th times the number of students $= \frac{5x}{6}$.

Therefore the total students $= \frac{x^2}{6} + \frac{5x}{6} + 10$.

Hence, the polynomial will be $p(x) = \frac{x^2}{6} + \frac{5x}{6} + 10$.

So, option (a) is correct.

2. (b) $\frac{5}{6}$ is the coefficient of $x$.

So, option (b) is correct.

3. (c) $\frac{1}{6}$ is the coefficient of $x^2$.

So, option (c) is correct.

4. (a) Let $p(x) = \frac{x^2}{6} + \frac{5x}{6} + 10$

At $x = 2$,

$$p(2) = \frac{(2)^2}{6} + \frac{5 \times 2}{6} + 10$$

$$= \frac{4}{6} + \frac{10}{6} + 10 = \frac{4 + 10 + 60}{6}$$

$$p(2) = \frac{74}{6} = \frac{37}{3}$$

So, option (a) is correct.

5. (b) When $p(x)$ is divided by $x$, then quotient is:

$$\frac{\frac{x^2}{6} + \frac{5}{6}x + 10}{x} = \frac{x}{6} + \frac{5}{6} + \frac{10}{x}$$

So, option (b) is correct.

## Case Study 2

Amit along with his four friends visited the house of Rohit, who was a common friend. There they meet his father, who was having keen interest in mathematics. Rohit's father wanted to test the practical knowledge of all his friends, so he showed some objects like a cuboid shaped geometry box, a rectangular photo frame, a circular cardboard, square shaped files and a cube. He started asking the following questions one by one.

(Cuboid Geometry Box)


(Rectangular Photo Frame)


(Circular Cardboard)

*On the basis of the above information, solve the following questions:*

**Q 1. If the area of circular cardboard is $49\pi x^2 + 70\pi x + 25\pi$, what is the radius of this object?**

a. $(7x + 5)$  b. $\pi(7x + 5)$
c. $-5/7$  d. $7/5$

**Q 2. If the volume of geometry box is $x^3 - 2x^2 - x + 2$, then the possible dimensions of this box are:**

a. $(x + 1), (x + 1), (x + 2)$ b. $(x + 1), (x - 1), (x + 2)$
c. $(x - 1), (x + 1), (x - 2)$ d. $(x - 1), (x - 1), (x + 2)$

**Q 3. If the area of a file is $4x^2 + 4x + 1$, what is the perimeter of this file?**

a. $2x + 1$  b. $4x + 1$
c. $4(2x + 1)$  d. $(8x + 2)$

**Q 4. If the area of rectangular photo frame is $12x^2 - 7x + 1$, what are the possible dimensions of photo frame?**

a. $(3x - 1), (4x - 1)$  b. $(3x + 1), (4x + 1)$
c. $(3x - 1), (4x + 1)$  d. $(3x + 1), (4x - 1)$

**Q 5. If the volume of cube is $8a^3 - b^3 - 12a^2b + 6ab^2$, what is the side of cube?**

a. $(2a + b)$  b. $(2a - b)$
c. $(2a + 3b)$  d. $(3a - 2b)$

## Solutions

1. (a) Given,

Area of circular cardboard $= 49\pi x^2 + 70\pi x + 25\pi$

We know that,

Area of circle $= \pi r^2$

$\therefore \quad \pi r^2 = \pi(49x^2 + 70x + 25)$

$\Rightarrow \quad r^2 = (49x^2 + 35x + 35x + 25)$

$= [7x(7x + 5) + 5(7x + 5)]$

$= [(7x + 5)^2] = (7x + 5)(7x + 5)$

$\therefore \quad r = 7x + 5$

Hence, the radius of the circle is $(7x + 5)$.

So, option (a) is correct.

2. (c) Volume $= x^3 - 2x^2 - x + 2$

$= x^3 - x^2 - x^2 + x - 2x + 2$

$= x^2(x - 1) - x(x - 1) - 2(x - 1)$

$= (x^2 - x - 2)(x - 1)$

$= (x - 1)(x^2 - 2x + x - 2)$

$= (x - 1)[x(x - 2) + 1(x - 2)]$

$= (x - 1)(x - 2)(x + 1)$

Hence, possible dimensions are $(x - 1), (x + 1), (x - 2)$.

So, option (c) is correct.

3. (c) We have, area of a square shape file $= 4x^2 + 4x + 1$

We know that,

Area of square $= (\text{Side})^2$

$\therefore \quad (\text{Side})^2 = (4x^2 + 4x + 1) = (2x + 1)^2$

$[\because (a + b)^2 = a^2 + b^2 + 2ab]$

$\therefore \quad$ Perimeter $= 4 \times (2x + 1) = 4(2x + 1)$

So, option (c) is correct.

4. (a) Area of rectangle $= l \times b = 12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (4x - 1)(3x - 1)$

Hence, possible dimensions are $(4x - 1), (3x - 1)$.

So, option (a) is correct.

5. (b) Volume of cube $= 8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - (b)^3 - 6ab(2a - b)$

$= (2a - b)\,[4a^2 + 2ab + b^2] - 6ab\,[2a - b]$

$= (2a - b)\,[4a^2 - 4ab + b^2]$

$= (2a - b)\,(2a - b)^2$

$= (2a - b)^3$ [$\because$ volume of cube = (side)$^3$]

Hence, the side of cube is $(2a - b)$.

So, option (b) is correct.

## Case Study 3

Nari Niketan is an organisation to help the women and child having distress. Swati donated some amount to this organisation for betterment. The amount of donation is represented by the expression ₹ $4x^2 + \frac{1}{4x^2}$. She also discussed her friends about this organisation. Some of her friends wanted to know the amount of donation, but she did not disclose this amount to anyone. Some how her friend got to know that she gave amount having expression $\left(2x + \frac{1}{2x}\right)$, whose value is ₹ 90.

*On the basis of the above information, solve the following questions:*

**Q 1. The amount donated by Swati in the expression form is:**

a. linear equation b. quadratic equation
c. algebraic expression d. polynomial

**Q 2. If $x = \sqrt{2}$, then the amount donated by Swati is:**

a. ₹ 8 b. ₹ 8.125 c. ₹ 8.75 d. ₹ 9

**Q 3. The value of $\left(2x + \frac{1}{2x}\right)^2$ is:**

a. 8000 b. 8100 c. 8200 d. 8300

**Q 4. The amount donated by Swati (in ₹) is:**

a. ₹ 9020 b. ₹ 8096 c. ₹ 8090 d. ₹ 9000

**Q 5. If $x = 5$, then the value of donated expression is:**

a. $\frac{10001}{100}$ b. $\frac{1003}{100}$ c. $\frac{999}{100}$ d. $\frac{10005}{100}$

### Solutions

1. (c) The amount donated by Swati in the expression form is algebraic expression.

So, option (c) is correct.

2. (b) Since, amount donated by Swati is $4x^2 + \frac{1}{4x^2}$.

At $x = \sqrt{2}$, then

$$4x^2 + \frac{1}{4x^2} = 4(\sqrt{2})^2 + \frac{1}{4(\sqrt{2})^2} = 8 + \frac{1}{8}$$

$= 8 + 0.125 =$ ₹ 8.125

So, option (b) is correct.

3. (b) Given, $\left(2x + \frac{1}{2x}\right) = 90$

Squaring both sides, we get

$$\left(2x + \frac{1}{2x}\right)^2 = (90)^2$$

$= 8100$

So, option (b) is correct.

4. (b) Now, $4x^2 + \frac{1}{4x^2} = (2x)^2 + \frac{1}{(2x)^2} + 4 - 4$

$$= \left(2x + \frac{1}{2x}\right)^2 - 4$$

$= (90)^2 - 4 = 8100 - 4 =$ ₹ 8096

Hence, amount donated by Swati is ₹ 8096.

So, option (b) is correct.

5. (a) Given, $x = 5$

$$\therefore \left(4x^2 + \frac{1}{4x^2}\right) = 4(5)^2 + \frac{1}{4(5)^2}$$

$$= 4 \times 25 + \frac{1}{4 \times 25}$$

$$= 100 + \frac{1}{100} = \frac{10001}{100}$$

So, option (a) is correct.

## Case Study 4

In the current scenario, people use such door whose top half part is made of glass and bottom half part is wooden.

The glass portion of the door is having length and width in the ratio of 5 : 3. The wooden frame around the glass portion adds 11 inches to the total width and 14 inches to the total length. Consider the length of the glass portion as $5x$ inches:

*On the basis of the above information, solve the following questions:*

**Q 1. Find the total length of the glass portion of the door (in inches) is represented in terms of $x$.**

**Q 2. Find the total width of the glass portion of the door (in inches).**

**Q 3. Write the polynomial representation of the area top half part of the door.**

**Q 4. Find the zeroes of the polynomial representing the area.**

## Solutions

1. The total length of the glass portion in the door is represented by $(5x + 14)$ inches.
2. The total width of the glass portion in the door is $(3x + 11)$ inches.
3. The area of top half part of the door

$$= \text{length} \times \text{width}$$
$$= (5x + 14)(3x + 11)$$
$$= 15x^2 + 55x + 42x + 154$$
$$= 15x^2 + 97x + 154$$

4. We have area, $p(x) = (5x + 14)(3x + 11)$

For finding zeroes, put $p(x) = 0$

$\therefore (5x + 14)(3x + 11) = 0$

$\Rightarrow (5x + 14) = 0$ or $(3x + 11) = 0$

$\Rightarrow x = \frac{-14}{5}$ or $x = \frac{-11}{3}$

## Case Study 5

A teacher told 10 students to write a polynomial on the blackboard.

The students wrote the following polynomials:

| | |
|---|---|
| 1. $\sqrt{5}x^3 + 1$ | 2. $20x^3 + 3x + 8$ |
| 3. $x - 2$ | 4. $x^2 + \frac{12x}{35} + \frac{1}{35}$ |
| 5. $3x^3 - 4x^2 - 12x + 16$ | 6. $-2x - 5$ |
| 7. $\frac{\pi}{2}x^2 + x$ | 8. $9x^2 - 361$ |
| 9. $\frac{12}{x} - 64 - 3x^2 + 24x$ | 10. $8x^3$ |

*On the basis of the above information, solve the following questions:*

**Q 1. How many students wrote quadratic polynomial?**

**Q 2. How many students wrote a binomial?**

**Q 3. Find the zeroes of the polynomial $p(x) = -2x - 5$.**

**Q 4. Factorise: $9x^2 - 361$.**

## Solutions

1. There are 3 students to write a quadratic polynomial on the blackboard.

i.e, $x^2 + \frac{12x}{35} + \frac{1}{35}, \frac{\pi}{2}x^2 + x, 9x^2 - 361$

2. There are 5 students to write a binomial on the blackboard.

i.e, $\sqrt{5}x^3 + 1, x - 2, -2x - 5, \frac{\pi}{2}x^2 + x, 9x^2 - 361$

3. Consider $p(x) = 0$

$\Rightarrow -2x - 5 = 0$

$\Rightarrow x = \frac{-5}{2}$

So, $\frac{-5}{2}$ is a zero of the polynomial $(-2x - 5)$.

4. $9x^2 - 361 = (3x)^2 - (19)^2$ $[\because a^2 - b^2 = (a - b)(a + b)]$

$= (3x - 19)(3x + 19)$

## Very Short Answer Type Questions

**Q 1. Is $x^2 + \frac{4x^{3/2}}{\sqrt{x}}$ a polynomial? Justify your answer.**

**Q 2. What is the degree of the polynomial $(x^3 + 5)(4 - x^5)$?**

**Q 3. Write the zeroes of the polynomial $p(x) = x(x - 2)(x - 3)$.**

**Q 4. Find the value of $f(x) = 2x^2 + 7x + 3$ at $x = -2$.**

**Q 5. What is the degree of the polynomial $\frac{4x - 5x^2 + 6x^3}{2x}$?**

**Q 6. Find the remainder, when the polynomial $p(x) = 2x^3 - 2x^2 + 3x - 4$ is divided by $g(x) = x - 2$.**

**Q 7. Check whether $(5 + 2x)$ is a factor of $5x^3 + 7x$.**

**Q 8. Using factor theorem, show that $g(x) = x - 3$ is a factor of $p(x) = 2x^3 + 7x^2 - 24x - 45$.**

**Q 9. Factorise $125x^3 - 64y^3$.**

**Q 10. Factorise $- 6 + x + x^2$.**

**Q 11. Factorise $1 + 2ab - (a^2 + b^2)$.**

**Q 12. Write the coefficient of $x^2$ in the expansion of $(x - 2)^3$.**

**Q 13.** Expand $(-x+2y-3z)^2$.

**Q 14.** Evaluate $94 \times 96$ by using identity.

**Q 15.** Find the product $(x^2-1)(x^4+x^2+1)$.

## Short Answer Type-I Questions

**Q 1.** Identify the following types of polynomials on the basis of degree:

(i) $3x^2+5$ (ii) $z^3+4z+1$

(iii) $x^2+x$ (iv) $1+x$

**Q 2.** Write:

(i) the coefficient of $x$ in $\sqrt{5}-2\sqrt{3}x+7x^2$.

(ii) the constant term in $\frac{\pi}{2}x^2+8x-\frac{3}{11}\pi$.

**Q 3.** For the polynomial $\frac{x^3+2x+1}{5}-\frac{7}{2}x^2-x^6$, write:

(i) the degree of polynomial.

(ii) coefficient of $x^3$.

(iii) coefficient of $x^6$.

(iv) constant term.

**Q 4.** Verify that $-7$ and $\frac{3}{2}$ are the zeroes of the polynomial $p(x)=2x^2+11x-21$.

**Q 5.** If $x=\frac{-1}{2}$ is a zero of the polynomial $p(x)=8x^3-ax^2-x+2$, find the value of $a$.

**Q 6.** Suppose $p(x)=x^3-2x^2+x$ and $g(x)=(3-2x)$. Check whether $p(x)$ is a multiple of $g(x)$ or not.

**Q 7.** If the polynomials $(2x^3+ax^2+4x-5)$ and $(x^3+x^2-3x+a)$ leave the same remainder when divided by $(x-2)$, find the value of $a$.

**Q 8.** Find the values of $a$ and $b$, so that $ax^2+2x+b$ has $(x+1)$ and $(x-2)$ as factors.

**Q 9.** Expand $\left(\frac{1}{x}+\frac{y}{3}\right)^3$.

**Q 10.** If $x+\frac{1}{x}=7$, then find the value of $x^3+\frac{1}{x^3}$.

**Q 11.** Factorise $64a^3-27b^3-144a^2b+108ab^2$.

**Q 12.** If $a=3+b$, then what is the value of $a^3-b^3-9ab$?

**Q 13.** Factorise $x^4-125xy^3$.

**Q 14.** If $\sqrt{m}+\sqrt{n}-\sqrt{p}=0$, then find the value of $(m+n-p)^2$.

**Q 15.** Evaluate $(\sqrt{2}+\sqrt{3})^2+(\sqrt{5}-\sqrt{2})^2$.

## Short Answer Type-II Questions

**Q 1.** Which of the following expressions are polynomials? In case of a polynomial, write its degree:

(i) $y^3+\sqrt{3}y$ (ii) $\frac{2}{3}x^2-\frac{7}{4}x+9$

(iii) $\frac{1}{\sqrt{2}}t^2-\sqrt{2}t+2$ (iv) $2u^3-3u^2+\sqrt{u}-1$

(v) $l^{100}-1$ (vi) $m^4-m^{3/2}+m-1$

**Q 2.** If $f(x)=x^2-5x+7$, evaluate $f(2)-f(-1)+f\left(\frac{1}{3}\right)$.

**Q 3.** If $x=0$ and $x=-1$ are the zeroes of the polynomial $f(x)=2x^3-3x^2+ax+b$, find the value of $a$ and $b$.

**Q 4.** Factorise $5\sqrt{5}x^2+30x+8\sqrt{5}$ by splitting the middle term.

**Q 5.** Without actual division, show that $(x^3-3x^2-13x+15)$ is exactly divisible by $(x^2+2x-3)$.

**Q 6.** If $3x+2y=12$ and $xy=6$, find the value of $27x^3+8y^3$.

**Q 7.** Factorise $2(x+y)^2-9(x+y)-5$ by splitting the middle term.

**Q 8.** Factorise:

(i) $a^3-0.216$ (ii) $x^6-7x^3-8$

**Q 9.** Factorise $(x-y)^2-7(x^2-y^2)+12(x+y)^2$.

**Q 10.** Factorise $9x^2+y^2+z^2-6xy+2yz-6xz$. Hence, find its value when $x=1$, $y=2$ and $z=-1$.

**Q 11.** Simplify the following:

(i) $(7a-5b)(49a^2+35ab+25b^2)$

(ii) $(6m-n)(36m^2+6mn+n^2)-(3m+2n)^3$

**Q 12.** Using identities, evaluate the following:

(i) $53 \times 47$ (ii) $101^3$

(iii) $105^2$

**Q 13.** If $z^2+\frac{1}{z^2}=34$, find the value of $z^3+\frac{1}{z^3}$, using only the positive value of $z+\frac{1}{z}$.

**Q 14.** Simplify $[(x+y)^3-(x-y)^3-6y(x^2-y^2)]$.

**Q 15.** If $a^2+b^2+c^2=20$ and $a+b+c=0$, find $ab+bc+ca$.

**Q 16.** If $x+\frac{1}{x}=3$, find the value of $x^4+\frac{1}{x^4}$.

## Long Answer Type Questions

**Q 1.** If $p(x) = x^3 + 2x^2 - 5x - 6$, then find $p(2)$, $p(-1)$, $p(-3)$ and $p(0)$. What do you conclude about the zeroes of $p(x)$? Is 0 a zero of $p(x)$?

**Q 2.** Show that $\frac{1}{3}$ and $\frac{4}{3}$ are zeroes of the polynomial $9x^3 - 6x^2 - 11x + 4$. Also, find the third zero of the polynomial.

**Q 3.** Factorise $x^3 + 6x^2 + 11x + 6$.

**Q 4.** Factorise $2x^3 - 3x^2 - 17x + 30$.

**Q 5.** Simplify:

$$(x+y+z)^2 + \left(x + \frac{y}{2} + \frac{z}{3}\right)^2 - \left(\frac{x}{2} + \frac{y}{3} + \frac{z}{4}\right)^2$$

**Q 6.** Factorise:

(i) $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$

(ii) $27p^3 - \frac{9}{2}p^2 + \frac{1}{4}p - \frac{1}{216}$

**Q 7.** Simplify:

(i) $\left(\frac{x}{2} + \frac{y}{3}\right)^3 - \left(\frac{x}{2} - \frac{y}{3}\right)^3$

(ii) $\frac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 + 155 \times 55 + 55 \times 55}$

**Q 8.** Factorise:

(i) $2a^7 - 128a$  (ii) $a^3 - b^3 - a + b$

**Q 9.** If $x = -2$ and $y = 1$, by using an identity, find the value of the following:

(i) $(4y^2 - 9x^2)(16y^4 + 36x^2y^2 + 81x^4)$

(ii) $\left(5y + \frac{15}{y}\right)\left(25y^2 - 75 + \frac{225}{y^2}\right)$

**Q 10.** Factorise each of the following expressions:

(i) $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$  (ii) $x^2 + 6\sqrt{2}x + 10$

**Q 11.** Factorise:

(i) $\frac{1}{81}a^2 + \frac{1}{9}b^2 + 100 - \frac{2}{27}ab - \frac{20}{3}b + \frac{20}{9}a$

(ii) $8(a+1)^2 - 2(a+1)(b+2) - 15(b+2)^2$

## Solutions

### Very Short Answer Type Questions

1. Yes, $x^2 + \frac{4x^{3/2}}{\sqrt{x}} = x^2 + 4x^{3/2} \times x^{-\frac{1}{2}}$

$= x^2 + 4x^{\frac{3}{2} - \frac{1}{2}} = x^2 + 4x$

which is a polynomial.

2. Degree of $(x^3 + 5) = 3$ and that of $(4 - x^5) = 5$

$\therefore$ Degree of $(x^3 + 5)(4 - x^5) = 3 + 5 = 8$

3. For zero of the polynomial $p(x)$, put $p(x) = 0$

$\therefore \quad x(x-2)(x-3) = 0 \quad \Rightarrow x = 0, 2, 3$

4. Given, $f(x) = 2x^2 + 7x + 3$

At $x = -2$,

$f(-2) = 2(-2)^2 + 7(-2) + 3$

$= 8 - 14 + 3 = 11 - 14 = -3$

5. $\frac{4x - 5x^2 + 6x^3}{2x} = \frac{4x}{2x} - \frac{5x^2}{2x} + \frac{6x^3}{2x}$

$= 2 - \frac{5x}{2} + 3x^2$

$\therefore$ Degree of $\frac{4x - 5x^2 + 6x^3}{2x} = 2$

6.

**TR!CK**

*By using remainder theorem, when $p(x)$ is divided by $(x - a)$, then remainder is $p(a)$.*

When $p(x) = 2x^3 - 2x^2 + 3x - 4$ is divided by $g(x) = x - 2$, then remainder is

$p(2) = 2(2)^3 - 2(2)^2 + 3(2) - 4$

$= 16 - 8 + 6 - 4$

$= 10$

7. Let $p(x) = 5x^3 + 7x$

Consider $5 + 2x = 0 \quad \Rightarrow \quad x = \frac{-5}{2}$

$\therefore p\left(\frac{-5}{2}\right) = 5\left(\frac{-5}{2}\right)^3 + 7\left(\frac{-5}{2}\right)$

$= -\frac{625}{8} - \frac{35}{2} = \frac{-625 - 140}{8}$

$= -\frac{765}{8} \neq 0.$

Hence, $(5 + 2x)$ is not a factor of $5x^3 + 7x$.

8. Given, $p(x) = 2x^3 + 7x^2 - 24x - 45$ and $g(x) = x - 3$.

Consider $g(x) = x - 3 = 0$

$\Rightarrow \quad x = 3$

$\therefore \quad p(3) = 2(3)^3 + 7(3)^2 - 24(3) - 45$

$= 54 + 63 - 72 - 45$

$= 117 - 117 = 0$

Hence, $g(x)$ is a factor of $p(x)$.

9. $125x^3 - 64y^3 = (5x)^3 - (4y)^3$

By using identity, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$, we get

$125x^3 - 64y^3 = (5x - 4y)(25x^2 + 20xy + 16y^2)$

10. $-6 + x + x^2 = x^2 + x - 6$

$= x^2 + 3x - 2x - 6$

$= x(x+3) - 2(x+3)$

$= (x+3)(x-2)$

11. $1 + 2ab - (a^2 + b^2) = 1 - (a^2 + b^2 - 2ab)$

$= (1)^2 - (a-b)^2$

$= [1 - (a-b)][1 + (a-b)]$

$[\because a^2 - b^2 = (a-b)(a+b)]$

$= (1 - a + b)(1 + a - b)$

12. $(x-2)^3 = x^3 - 3 \times x^2 \times 2 + 3 \times x \times 2^2 - 2^3$

$= x^3 - 6x^2 + 12x - 8$

$[\because (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3]$

Hence, the coefficient of $x^2$ is $-6$.

13. $(-x + 2y - 3z)^2 = (-x)^2 + (2y)^2 + (-3z)^2 + [2 \times -x \times 2y] + [2 \times 2y \times -3z] + [2 \times -3z \times -x]$

$[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac]$

$= x^2 + 4y^2 + 9z^2 - 4xy - 12yz + 6xz$

14. $94 \times 96 = (95 - 1) \times (95 + 1) = (95)^2 - (1)^2$

$[\because (x-y)(x+y) = x^2 - y^2]$

$= 9025 - 1 = 9024$

15. Using identity, $(a-b)(a^2 + ab + b^2) = a^3 - b^3$

Here, $a = x^2$ and $b = 1$

$\Rightarrow (x^2 - 1)(x^4 + x^2 + 1)$

$= (x^2 - 1)[(x^2)^2 + (x^2 \times 1) + (1)^2]$

$= (x^2)^3 - (1)^3 = (x^6 - 1)$

## Short Answer Type-I Questions

1. (i) Degree of polynomial $3x^2 + 5$ is 2. Hence, it is a quadratic polynomial.

(ii) Degree of polynomial $z^3 + 4z + 1$ is 3. Hence, it is a cubic polynomial.

(iii) Degree of polynomial $x^2 + x$ is 2. Hence, it is a quadratic polynomial.

(iv) Degree of polynomial $1 + x$ is 1. Hence, it is a linear polynomial.

2. (i) The coefficient of $x$ is $-2\sqrt{3}$.

(ii) The constant term is $\frac{-3}{11}\pi$.

3. Given, $p(x) = \frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$

$= \frac{1}{5}x^3 + \frac{2}{5}x + \frac{1}{5} - \frac{7}{2}x^2 - x^6$

(i) Degree of polynomial is highest power of variable, *i.e.*, 6.

(ii) Coefficient of $x^3$ in given polynomial is $\frac{1}{5}$.

(iii) Coefficient of $x^6$ in given polynomial is $-1$.

(iv) Constant term in given polynomial is $\frac{1}{5}$.

4. Given, $p(x) = 2x^2 + 11x - 21$

$p(-7) = 2(-7)^2 + 11(-7) - 21 = 2 \times 49 - 77 - 21$

$= 98 - 98 = 0$

$p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^2 + 11\left(\frac{3}{2}\right) - 21 = 2 \times \frac{9}{4} + \frac{33}{2} - 21$

$= \frac{9}{2} + \frac{33}{2} - 21 = \frac{9 + 33 - 42}{2} = 0$

Since, $p(-7) = 0$ and $p\left(\frac{3}{2}\right) = 0$

$\therefore -7$ and $\frac{3}{2}$ are the zeroes of the polynomial $p(x)$.

5. If $\frac{-1}{2}$ is a zero of $p(x)$, then

$p\left(\frac{-1}{2}\right) = 0$

$\Rightarrow 8\left(\frac{-1}{2}\right)^3 - a\left(\frac{-1}{2}\right)^2 - \left(\frac{-1}{2}\right) + 2 = 0$

$\Rightarrow 8 \times \frac{-1}{8} - a \times \frac{1}{4} + \frac{1}{2} + 2 = 0$

$\Rightarrow -1 - \frac{a}{4} + \frac{5}{2} = 0$

$\Rightarrow \frac{a}{4} = \frac{3}{2}$

$\Rightarrow a = 3 \times 2$

$\Rightarrow a = 6$

6. Consider $g(x) = 0 \Rightarrow 3 - 2x = 0$

$\Rightarrow x = \frac{3}{2}$

By using remainder theorem, when $p(x)$ is divided by $(3 - 2x)$, then the remainder is $p\left(\frac{3}{2}\right)$.

Given, $p(x) = x^3 - 2x^2 + x$

$\therefore p\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^3 - 2\left(\frac{3}{2}\right)^2 + \frac{3}{2}$

$= \frac{27}{8} - \frac{9}{2} + \frac{3}{2}$

$= \frac{27}{8} - \frac{6}{2} = \frac{27 - 24}{8} = \frac{3}{8} \neq 0$

Thus, when $p(x)$ is divided by $g(x)$, the remainder is non zero.

Hence, $p(x)$ is not a multiple of $g(x)$.

7. Let $g(x) = 2x^3 + ax^2 + 4x - 5$ and
$h(x) = x^3 + x^2 - 3x + a$

When $g(x)$ and $h(x)$ are divided by $(x - 2)$, then remainders are

$$g(2) = 2(2)^3 + a(2)^2 + 4(2) - 5$$
$$= 16 + 4a + 8 - 5$$
$$= 19 + 4a$$

and
$$h(2) = (2)^3 + (2)^2 - 3(2) + a$$
$$= 8 + 4 - 6 + a$$
$$= 6 + a$$

According to the given condition,

$$g(2) = h(2)$$

$\therefore$ $19 + 4a = 6 + a \Rightarrow 3a = -13$

$\Rightarrow$ $a = \frac{-13}{3}$

8. Let $p(x) = ax^2 + 2x + b$

Since, $(x + 1)$ and $(x - 2)$ are the factors of $p(x)$.

$\therefore$ $p(-1) = 0$ and $p(2) = 0$

$\Rightarrow a(-1)^2 + 2(-1) + b = 0$

and $a(2)^2 + 2(2) + b = 0$

$\Rightarrow$ $a - 2 + b = 0$

and $4a + 4 + b = 0$

$\Rightarrow$ $a + b = 2$

and $4a + b = -4$

On solving, we get

$a = -2$ and $b = 4$

9.

**TR!CK**

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$\left(\frac{1}{x}+\frac{y}{3}\right)^3 = \left(\frac{1}{x}\right)^3 + \left(\frac{y}{3}\right)^3 + 3\left(\frac{1}{x}\right)\left(\frac{y}{3}\right)\left(\frac{1}{x}+\frac{y}{3}\right)$$
$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x}\left(\frac{1}{x}+\frac{y}{3}\right)$$
$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x}$$

10. Given, $x + \frac{1}{x} = 7$ ...(1)

Cubing both sides, we get

$$\left(x+\frac{1}{x}\right)^3 = 7^3$$

$\Rightarrow$ $x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 343$

$\Rightarrow$ $x^3 + \frac{1}{x^3} + 3 \times 7 = 343$ [From eq. (1)]

$\Rightarrow$ $x^3 + \frac{1}{x^3} = 343 - 21 = 322$

Hence, the value of $x^3 + \frac{1}{x^3}$ is 322.

11. $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$$= (4a)^3 - (3b)^3 - 3 \times (4a)^2 \times (3b) + 3 \times (4a) \times (3b)^2$$
$$= (4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b)$$
$$= (4a - 3b)^3 \quad [\because (x-y)^3 = (x)^3 - (y)^3 - 3xy(x-y)]$$

12. Given, $a = 3 + b$

$\Rightarrow$ $a - b = 3$ ...(1)

Cubing both sides, we get

$$(a - b)^3 = (3)^3$$

$\Rightarrow$ $a^3 - b^3 - 3ab(a - b) = 27$

$\Rightarrow$ $a^3 - b^3 - 3ab \times 3 = 27$ [From eq. (1)]

$\Rightarrow$ $a^3 - b^3 - 9ab = 27$

Hence, the value of given expression is 27.

13. $x^4 - 125xy^3 = x(x^3 - 125y^3)$
$= x[(x)^3 - (5y)^3]$

**TR!CK**

$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$= x(x - 5y)[x^2 + (5y)^2 + x \times 5y]$$
$$= x(x - 5y)(x^2 + 25y^2 + 5xy)$$

14. Given, $\sqrt{m} + \sqrt{n} - \sqrt{p} = 0$

$\Rightarrow$ $\sqrt{m} + \sqrt{n} = \sqrt{p}$

Squaring both sides, we get

$$(\sqrt{m} + \sqrt{n})^2 = (\sqrt{p})^2$$

$\Rightarrow$ $m + n + 2\sqrt{m}\sqrt{n} = p$

$\Rightarrow$ $m + n - p = -2\sqrt{mn}$

Again, squaring both sides, we get

$$(m + n - p)^2 = 4mn$$

15. $(\sqrt{2} + \sqrt{3})^2 + (\sqrt{5} - \sqrt{2})^2$

$$= (\sqrt{2})^2 + (\sqrt{3})^2 + 2 \times \sqrt{2} \times \sqrt{3} + (\sqrt{5})^2 + (\sqrt{2})^2 - 2 \times (\sqrt{5}) \times (\sqrt{2})$$

**TR!CKS**

- $(a + b)^2 = a^2 + b^2 + 2ab$
- $(a - b)^2 = a^2 + b^2 - 2ab$

$$= 2 + 3 + 2\sqrt{6} + 5 + 2 - 2\sqrt{10}$$
$$= 12 + 2\sqrt{6} - 2\sqrt{10}$$
$$= 2(6 + \sqrt{6} - \sqrt{10})$$

## Short Answer Type-II Questions

1. (i) $y^3 + \sqrt{3}y$ is a polynomial.

The highest power of $y$ is 3.
So, it is a polynomial of degree 3.

(ii) $\frac{2}{3}x^2 - \frac{7}{4}x + 9$ is a polynomial.

The highest power of $x$ is 2.
So, it is a polynomial of degree 2.

(iii) $\frac{1}{\sqrt{2}}t^2 - \sqrt{2}t + 2$ is a polynomial.

The highest power of $t$ is 2.
So, it is a polynomial of degree 2.

(iv) $2u^3 - 3u^2 + \sqrt{u} - 1$ is not a polynomial as $\sqrt{u}$ (written as $u^{1/2}$) has rational power of $u$.

(v) $l^{100} - 1$ is a polynomial.

The highest power of $l$ is 100.
So, it is a polynomial of degree 100.

(vi) $m^4 - m^{3/2} + m - 1$ is not a polynomial as $m^{3/2}$ has the exponent of $m$ is $\frac{3}{2}$, which is rational.

2. Given, $f(x) = x^2 - 5x + 7$

when $x = 2, f(2) = (2)^2 - 5(2) + 7$
$= 4 - 10 + 7 = 1$

when $x = -1, f(-1) = (-1)^2 - 5(-1) + 7$
$= 1 + 5 + 7 = 13$

and when $x = \frac{1}{3}, f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^2 - 5\left(\frac{1}{3}\right) + 7$
$= \frac{1}{9} - \frac{5}{3} + 7 = \frac{1-15+63}{9} = \frac{49}{9}$

Now, $f(2) - f(-1) + f\left(\frac{1}{3}\right)$

$= 1 - 13 + \frac{49}{9} = -12 + \frac{49}{9} = \frac{-59}{9}$

3. Given that, $f(x) = 2x^3 - 3x^2 + ax + b$

$\therefore$ $f(0) = 2(0)^3 - 3(0)^2 + a(0) + b$
$= 0 - 0 + 0 + b = b$

and $f(-1) = 2(-1)^3 - 3(-1)^2 + a(-1) + b$
$= -2 - 3 - a + b$
$= -5 - a + b$

Since, $x = 0$ and $x = -1$ are the zeroes of the polynomial $f(x)$.

$\therefore$ $f(0) = 0$ and $f(-1) = 0$
$\Rightarrow$ $b = 0$
and $-5 - a + b = 0$
$\Rightarrow$ $-5 - a + 0 = 0$
$\Rightarrow$ $a = -5$

Hence, $a = -5$ and $b = 0$.

4. $5\sqrt{5}x^2 + 30x + 8\sqrt{5}$

(Here, product, $ac = 200$ and sum, $b = 30 = 20 + 10$)

$= 5\sqrt{5}x^2 + 20x + 10x + 8\sqrt{5}$
$= 5\sqrt{5}x^2 + 20x + 2\sqrt{5}\sqrt{5}x + 8\sqrt{5}$
$= 5x(\sqrt{5}x + 4) + 2\sqrt{5}(\sqrt{5}x + 4)$
$= (\sqrt{5}x + 4)(5x + 2\sqrt{5})$
$= \sqrt{5}(\sqrt{5}x + 2)(\sqrt{5}x + 4)$

5. Let $p(x) = x^3 - 3x^2 - 13x + 15$
and $g(x) = x^2 + 2x - 3$
$= x^2 + 3x - x - 3$
$= x(x + 3) - 1(x + 3)$
$= (x - 1)(x + 3)$

Consider $g(x) = 0 \Rightarrow (x - 1)(x + 3) = 0$
$\Rightarrow$ $x = 1, -3.$

Without actual division, we have to show that $p(x)$ is exactly divisible by $g(x)$.
Thus, we saw that $p(x)$ is divisible by $(x - 1)$ and $(x + 3)$.
*i.e.* we will show that $p(1) = 0$ and $p(-3) = 0$

Now, $p(1) = (1)^3 - 3(1)^2 - 13(1) + 15$
$= 1 - 3 - 13 + 15$
$= 0$

and $p(-3) = (-3)^3 - 3(-3)^2 - 13(-3) + 15$
$= -27 - 27 + 39 + 15$
$= 0$ **Hence proved.**

6. Given, $3x + 2y = 12$ ...(1)

Cubing both sides, we get
$(3x + 2y)^3 = 12^3$
$\Rightarrow$ $(3x)^3 + (2y)^3 + 3 \times 3x \times 2y\,(3x + 2y) = 1728$
$\Rightarrow$ $27x^3 + 8y^3 + 18xy\,(3x + 2y) = 1728$
$\Rightarrow$ $27x^3 + 8y^3 + 18 \times 6 \times 12 = 1728$
[From eq. (1) and $xy = 6$]
$\Rightarrow$ $27x^3 + 8y^3 + 1296 = 1728$
$\Rightarrow$ $27x^3 + 8y^3 = 1728 - 1296 = 432$

Hence, the value of $27x^3 + 8y^3$ is 432.

7. We have, $2(x + y)^2 - 9(x + y) - 5$
$= 2l^2 - 9l - 5$, where $x + y = l$

Here, product $= 10$ and sum $= -9$
$= 2l^2 - 10l + l - 5$
$= 2l(l - 5) + 1(l - 5)$
$= (2l + 1)(l - 5)$
$= [2(x+y)+1][x+y-5] = (2x+2y+1)(x+y-5)$

8. (i) $a^3 - 0.216 = (a)^3 - (0.6)^3$

**TR!CK**
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (a - 0.6)\{(a)^2 + a \times 0.6 + (0.6)^2\}$
$= (a - 0.6)(a^2 + 0.6a + 0.36)$

(ii) $x^6 - 7x^3 - 8 = t^2 - 7t - 8$, where $x^3 = t$
$= t^2 - 8t + t - 8$
$= t(t - 8) + 1(t - 8)$
$= (t - 8)(t + 1)$
$= (x^3 - 8)(x^3 + 1)$ [Put $t = x^3$]
$= [(x)^3 - (2)^3][(x)^3 + (1)^3]$
$= (x - 2)(x^2 + 2x + 4)(x + 1)(x^2 - x + 1)$
$= (x - 2)(x + 1)(x^2 + 2x + 4)(x^2 - x + 1)$

9. $(x-y)^2 - 7(x^2-y^2) + 12(x+y)^2$

$= (x-y)^2 - 7(x-y)(x+y) + 12(x+y)^2$

$= (x-y)^2 - 4(x+y)(x-y) - 3(x+y)(x-y) + 12(x+y)^2$

$= (x-y)(x-y-4x-4y) - 3(x+y)(x-y-4x-4y)$

$= (x-y)(-3x-5y) - 3(x+y)(-3x-5y)$

$= -(3x+5y)(x-y-3x-3y)$

$= -(3x+5y)(-2x-4y)$

$= 2(x+2y)(3x+5y)$

10. $9x^2 + y^2 + z^2 - 6xy + 2yz - 6xz$

$= (-3x)^2 + (y)^2 + (z)^2 + 2\times(-3x)\times y + 2\times(y)\times(z) + 2\times(-3x)\times z$

$= (-3x+y+z)^2$

If $x = 1$, $y = 2$ and $z = -1$, then

$(-3x+y+z)^2 = [-3(1)+(2)+(-1)]^2$

$= (-3+2-1)^2 = (-2)^2 = 4$

11. (i) $(7a-5b)(49a^2+35ab+25b^2)$

$= (7a-5b)[(7a)^2 + 7a\times 5b + (5b)^2]$

$= (7a)^3 - (5b)^3$ $[\because (a-b)(a^2+ab+b^2) = a^3-b^3]$

$= 343a^3 - 125b^3$

(ii) $(6m-n)(36m^2+6mn+n^2) - (3m+2n)^3$

$= (6m-n)[(6m)^2 + 6m\times n + (n)^2] - (3m+2n)^3$

$= [(6m)^3 - (n)^3] - [(3m)^3 + (2n)^3 + 3\times 3m\times 2n(3m+2n)]$

**TR!CK**

$(a-b)(a^2+ab+b^2) = a^3-b^3$

$= (216m^3 - n^3) - (27m^3 + 8n^3 + 54m^2n + 36mn^2)$

$= 216m^3 - 27m^3 - n^3 - 8n^3 - 54m^2n - 36mn^2$

$= 189m^3 - 9n^3 - 54m^2n - 36mn^2$

12. (i) $53\times 47 = (50+3)(50-3)$

$= (50)^2 - (3)^2 = 2500 - 9 = 2491$

$[\because (a+b)(a-b) = a^2-b^2]$

(ii) $101^3 = (100+1)^3$

**TR!CK**

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$= (100)^3 + 1^3 + 3\times 100\times 1(100+1)$

$= 1000000 + 1 + 300\times 101$

$= 1000000 + 30301 = 1030301$

(iii) $(105)^2 = (100+5)^2$

$= (100)^2 + 5^2 + 2\times 100\times 5$

$[\because (a+b)^2 = a^2+b^2+2ab]$

$= 10000 + 25 + 1000 = 11025$

13. $\left(z+\frac{1}{z}\right)^2 = z^2 + \frac{1}{z^2} + 2$

$= 34 + 2 = 36$ $\left[\because z^2 + \frac{1}{z^2} = 34\right]$

$\therefore$ $z + \frac{1}{z} = \pm\sqrt{36} = \pm 6$

Taking only positive value of $z + \frac{1}{z}$, we get

$z + \frac{1}{z} = 6$

Now, $z^3 + \frac{1}{z^3} = \left(z+\frac{1}{z}\right)^3 - 3\left(z+\frac{1}{z}\right)$

$= 6^3 - 3\times 6 = 216 - 18 = 198$

14. $(x+y)^3 - (x-y)^3 - 6y(x^2-y^2)$

$= (x+y)^3 - (x-y)^3 - 6y(x+y)(x-y)$

$= (x+y)^3 - (x-y)^3 - 3(2y)(x+y)(x-y)$

$= (x+y)^3 - (x-y)^3 - 3(x+y)(x-y)[(x+y)-(x-y)]$

$= [(x+y)-(x-y)]^3$

**TR!CK**

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$= (x+y-x+y)^3 = (2y)^3 = 8y^3$

15. We know that,

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\Rightarrow$ $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow$ $(0)^2 = 20 + 2(ab+bc+ca)$

$\Rightarrow$ $2(ab+bc+ca) = -20$

$\Rightarrow$ $ab+bc+ca = \frac{-20}{2} = -10.$

16. We have, $x + \frac{1}{x} = 3$

Squaring both sides, we get

$\left(x+\frac{1}{x}\right)^2 = (3)^2 \Rightarrow x^2 + \frac{1}{x^2} + 2.x.\frac{1}{x} = 9$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 9 \Rightarrow x^2 + \frac{1}{x^2} = 7$

Again, squaring both sides, we get

$\left(x^2+\frac{1}{x^2}\right)^2 = 7^2 \Rightarrow x^4 + \frac{1}{x^4} + 2.x^2.\frac{1}{x^2} = 49$

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 49 \Rightarrow x^4 + \frac{1}{x^4} = 47$

## Long Answer Type Questions

1. Given, $p(x) = x^3 + 2x^2 - 5x - 6$

$\therefore$ $p(2) = (2)^3 + 2(2)^2 - 5(2) - 6$

$= 8 + 8 - 10 - 6 = 0$

$p(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$

$= -1 + 2 + 5 - 6 = 0$

$p(-3) = (-3)^3 + 2(-3)^2 - 5(-3) - 6$

$= -27 + 18 + 15 - 6$

$= -33 + 33 = 0$

$P(0) = (0)^3 + 2\times(0)^2 - 5(0) - 6$

$= 0 + 0 - 0 - 6 = -6 \neq 0$

Since, $p(2) = p(-1) = p(-3) = 0$, it shows that 2, –1 and –3 are the zeroes of $p(x)$.

But, $p(0) = -6 \neq 0$

$\therefore$ 0 is not a zero of $p(x)$.

2. Let $p(x) = 9x^3 - 6x^2 - 11x + 4$

When $x = \frac{1}{3}$, we have

$$p\left(\frac{1}{3}\right) = 9\left(\frac{1}{3}\right)^3 - 6\left(\frac{1}{3}\right)^2 - 11\left(\frac{1}{3}\right) + 4$$

$$= 9 \times \frac{1}{27} - 6 \times \frac{1}{9} - 11 \times \frac{1}{3} + 4$$

$$= \frac{1}{3} - \frac{2}{3} - \frac{11}{3} + 4 = \frac{1 - 2 - 11 + 12}{3} = \frac{0}{3} = 0$$

Thus, $x = \frac{1}{3}$ is a zero of polynomial $p(x)$.

When $x = \frac{4}{3}$, we have

$$p\left(\frac{4}{3}\right) = 9\left(\frac{4}{3}\right)^3 - 6\left(\frac{4}{3}\right)^2 - 11\left(\frac{4}{3}\right) + 4$$

$$= 9 \times \frac{64}{27} - 6 \times \frac{16}{9} - \frac{44}{3} + 4$$

$$= \frac{64}{3} - \frac{32}{3} - \frac{44}{3} + 4$$

$$= \frac{64 - 32 - 44 + 12}{3} = \frac{0}{3} = 0$$

Thus, $x = \frac{4}{3}$ is another zero of the polynomial $p(x)$.

Since $x = \frac{1}{3}$ and $x = \frac{4}{3}$ are the zeroes of $p(x)$.

Therefore, $\left(x - \frac{1}{3}\right)\left(x - \frac{4}{3}\right)$ or $\frac{(3x-1)}{3}\frac{(3x-4)}{3}$

or $\frac{1}{9}(9x^2 - 15x + 4)$ exactly divides $p(x)$.

We see that,

$$9x^3 - 6x^2 - 11x + 4 = 9x^3 - 15x^2 + 9x^2 + 4x - 15x + 4$$

$$= 9x^3 - 15x^2 + 4x + 9x^2 - 15x + 4$$

$$= x(9x^2 - 15x + 4) + 1(9x^2 - 15x + 4)$$

$$= (9x^2 - 15x + 4)(x + 1)$$

Hence, $x = -1$ is the third zero of the given polynomial.

3. Let $p(x) = x^3 + 6x^2 + 11x + 6$

The constant term in $p(x)$ is equal to 6 and the factors of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$.

Putting $x = -1$ in $p(x)$, we have

$$p(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$$

$$= -1 + 6 - 11 + 6 = 12 - 12 = 0$$

$\Rightarrow$ $p(-1) = 0$

$\therefore$ $(x + 1)$ is a factor of the polynomial $p(x)$.

We see that,

$$x^3 + 6x^2 + 11x + 6 = x^3 + x^2 + 5x^2 + 5x + 6x + 6$$

$$= x^2(x + 1) + 5x(x + 1) + 6(x + 1)$$

$$= (x + 1)(x^2 + 5x + 6)$$

$$\therefore p(x) = (x + 1)(x^2 + 5x + 6)$$

$$= (x + 1)(x^2 + 3x + 2x + 6)$$

$$= (x + 1)[x(x + 3) + 2(x + 3)]$$

$$= (x + 1)(x + 3)(x + 2)$$

Hence, $x^3 + 6x^2 + 11x + 6 = (x + 1)(x + 3)(x + 2)$

4. Let $f(x) = 2x^3 - 3x^2 - 17x + 30$

The constant term in $f(x)$ is 30 and the factors of 30 are $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6 \pm 10, \pm 15, \pm 30$.

Put $x = 2$ in $f(x)$, we have

$$f(2) = 2(2)^3 - 3(2)^2 - 17(2) + 30$$

$$= 16 - 12 - 34 + 30 = 4 - 4 = 0$$

$\therefore$ $(x - 2)$ is a factor of $f(x)$.

By using long division,

$$\begin{array}{r|l} x-2 & 2x^3 - 3x^2 - 17x + 30 \quad (2x^2 + x - 15 \\ & 2x^3 - 4x^2 \\ & \underline{- \quad +} \\ & x^2 - 17x \\ & x^2 - 2x \\ & \underline{- \quad +} \\ & -15x + 30 \\ & -15x + 30 \\ & \underline{+ \quad -} \\ & 0 \end{array}$$

$$\therefore f(x) = (x - 2)(2x^2 + x - 15)$$

$$= (x - 2)(2x^2 + 6x - 5x - 15)$$

$$= (x - 2)[2x(x + 3) - 5(x + 3)]$$

$$= (x - 2)(2x - 5)(x + 3)$$

Hence, $f(x) = (x - 2)(x + 3)(2x - 5)$.

5.

**TR!CK**

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$$\therefore (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$$

$$\left(x + \frac{y}{2} + \frac{z}{3}\right)^2 = x^2 + \left(\frac{y}{2}\right)^2 + \left(\frac{z}{3}\right)^2 + 2 \times x \times \frac{y}{2} + 2 \times \frac{y}{2} \times \frac{z}{3} + 2 \times x \times \frac{z}{3}$$

$$= x^2 + \frac{y^2}{4} + \frac{z^2}{9} + xy + \frac{yz}{3} + \frac{2xz}{3}$$

and $$\left(\frac{x}{2} + \frac{y}{3} + \frac{z}{4}\right)^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 + \left(\frac{z}{4}\right)^2 + 2 \times \frac{x}{2} \times \frac{y}{3} + 2 \times \frac{y}{3} \times \frac{z}{4} + 2 \times \frac{z}{4} \times \frac{x}{2}$$

$$= \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} + \frac{xy}{3} + \frac{yz}{6} + \frac{xz}{4}$$

$$\therefore (x + y + z)^2 + \left(x + \frac{y}{2} + \frac{z}{3}\right)^2 - \left(\frac{x}{2} + \frac{y}{3} + \frac{z}{4}\right)^2$$

$$= x^2 + y^2 + z^2 + 2xy + 2yz + 2xz + x^2 + \frac{y^2}{4} + \frac{z^2}{9} + xy + \frac{yz}{3} + \frac{2xz}{3} - \frac{x^2}{4} - \frac{y^2}{9} - \frac{z^2}{16} - \frac{xy}{3} - \frac{yz}{6} - \frac{xz}{4}$$

$$= \left(x^2 + x^2 - \frac{x^2}{4}\right) + \left(y^2 + \frac{y^2}{4} - \frac{y^2}{9}\right)$$
$$+ \left(z^2 + \frac{z^2}{9} - \frac{z^2}{16}\right) + \left(2xy + xy - \frac{xy}{3}\right)$$
$$+ \left(2yz + \frac{yz}{3} - \frac{yz}{6}\right) + \left(2xz + \frac{2xz}{3} - \frac{xz}{4}\right)$$
$$= \left(\frac{4x^2 + 4x^2 - x^2}{4}\right) + \left(\frac{36y^2 + 9y^2 - 4y^2}{36}\right)$$
$$+ \left(\frac{144z^2 + 16z^2 - 9z^2}{144}\right) + \left(\frac{6xy + 3xy - xy}{3}\right)$$
$$+ \left(\frac{12yz + 2yz - yz}{6}\right) + \left(\frac{24xz + 8xz - 3xz}{12}\right)$$
$$= \frac{7x^2}{4} + \frac{41y^2}{36} + \frac{151z^2}{144} + \frac{8xy}{3} + \frac{13yz}{6} + \frac{29xz}{12}$$

**6.** We have

(i) $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$

$$= 8p^3 + \frac{1}{125} + \frac{12}{5}p^2 + \frac{6}{25}p$$
$$= (2p)^3 + \left(\frac{1}{5}\right)^3 + \left(3 \times (2p)^2 \times \frac{1}{5}\right) + 3 \times 2p \times \left(\frac{1}{5}\right)^2$$

**TR!CK**

$a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3$

$$= \left(2p + \frac{1}{5}\right)^3 = \left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)$$

(ii) $27p^3 - \frac{9}{2}p^2 + \frac{1}{4}p - \frac{1}{216}$

$$= 27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$$
$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - \left(3 \times (3p)^2 \times \frac{1}{6}\right) + \left(3 \times 3p \times \left(\frac{1}{6}\right)^2\right)$$
$$= \left(3p - \frac{1}{6}\right)^3 \quad [\because a^3 - b^3 - 3a^2b + 3ab^2 = (a - b)^3]$$
$$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right).$$

**7.** (i) $\left(\frac{x}{2} + \frac{y}{3}\right)^3 - \left(\frac{x}{2} - \frac{y}{3}\right)^3$

$$= \left\{\left(\frac{x}{2}\right)^3 + \left(\frac{y}{3}\right)^3 + 3 \times \left(\frac{x}{2}\right)^2 \times \frac{y}{3} + 3 \times \frac{x}{2} \times \left(\frac{y}{3}\right)^2\right\}$$
$$- \left\{\left(\frac{x}{2}\right)^3 - \left(\frac{y}{3}\right)^3 - 3\left(\frac{x}{2}\right)^2 \times \frac{y}{3} + 3 \times \frac{x}{2} \times \left(\frac{y}{3}\right)^2\right\}$$

**TR!CK**

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$ and

$(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$$= \frac{x^3}{8} + \frac{y^3}{27} + \frac{x^2y}{4} + \frac{xy^2}{6} - \frac{x^3}{8} + \frac{y^3}{27} + \frac{x^2y}{4} - \frac{xy^2}{6}$$
$$= \frac{2y^3}{27} + \frac{x^2y}{2}.$$

(ii) $\frac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 + 155 \times 55 + 55 \times 55}$

$$= \frac{(155)^3 - (55)^3}{(155)^2 + 155 \times 55 + (55)^2}$$
$$= \frac{(155 - 55)[(155)^2 + 155 \times 55 + (55)^2]}{(155)^2 + 155 \times 55 + (55)^2}$$

**TR!CK**

$a^3 - b^3 = (a - b)(a^2 + ab + b)^2)$

$= 155 - 55 = 100$

**8.** (i) We have,

$(2a^7 - 128a)$
$= 2a \times (a^6 - 64) = 2a \times [(a^3)^2 - 8^2]$
$= 2a \times (a^3 - 8) \times (a^3 + 8) \quad [\because x^2 - y^2 = (x - y)(x + y)]$
$= 2a \times (a^3 - 2^3) \times (a^3 + 2^3)$
$= 2a \times [(a - 2) \times (a^2 + 2a + 4)] \times [(a + 2) \times (a^2 - 2a + 4]$

**TR!CK**

$a^3 - b^3 = (a - b)(a^2 + ab + b^2$

and $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= 2a(a - 2)(a + 2)(a^2 + 2a + 4)(a^2 - 2a + 4)$
$\therefore (2a^7 - 128a) = 2a(a - 2)(a + 2)(a^2 + 2a + 4)(a^2 - 2a + 4).$

(ii) We have,

$a^3 - b^3 - a + b$
$= (a^3 - b^3) - (a - b)$
$= (a - b)(a^2 + ab + b^2) - (a - b)$
$[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$
$= (a - b)(a^2 + ab + b^2 - 1).$
$\therefore (a^3 - b^3 - a + b) = (a - b)(a^2 + ab + b^2 - 1).$

**9.** (i) $(4y^2 - 9x^2)(16y^4 + 36x^2y^2 + 81x^4)$

$= (4y^2 - 9x^2)[(4y^2)^2 + 4y^2 \times 9x^2 + (9x^2)^2]$
$= (4y^2)^3 - (9x^2)^3$
$[\because (a - b)(a^2 + ab + b^2) = a^3 - b^3]$
$= 64y^6 - 729x^6$

If $x = -2$ and $y = 1$, then

$64y^6 - 729x^6 = 64(1)^6 - 729(-2)^6$
$= 64 - 729(64) = 64 - 46656 = -46592.$

(ii) $\left(5y+\frac{15}{y}\right)\left(25y^2-75+\frac{225}{y^2}\right)$

$$=\left(5y+\frac{15}{y}\right)\left((5y)^2-5y\times\frac{15}{y}+\left(\frac{15}{y}\right)^2\right)$$

$$=(5y)^3+\left(\frac{15}{y}\right)^3 \quad [\because (a+b)(a^2-ab+b^2)=a^3+b^3]$$

If $y=1$, then

$$(5y)^3+\left(\frac{15}{y}\right)^3=(5\times1)^3+\left(\frac{15}{1}\right)^3$$

$$=125+3375=3500.$$

**10.** (i) $4\sqrt{3}x^2+5x-2\sqrt{3}$

Here, product = – 24 and sum = 5

Clearly, 8 + (–3) = 5 and 8 × (–3) = – 24

$$\therefore\ 4\sqrt{3}x^2+5x-2\sqrt{3}=4\sqrt{3}x^2+8x-3x-2\sqrt{3}$$

$$=4x(\sqrt{3}x+2)-\sqrt{3}(\sqrt{3}x+2)$$

$$=(4x-\sqrt{3})(\sqrt{3}x+2).$$

(ii) $x^2+6\sqrt{2}x+10$

Here product = 10 and sum = $6\sqrt{2}$

Clearly, $5\sqrt{2}\times\sqrt{2}=10$ and $5\sqrt{2}+\sqrt{2}=6\sqrt{2}$

$$\therefore\quad x^2+6\sqrt{2}x+10$$

$$=x^2+5\sqrt{2}x+\sqrt{2}x+10$$

$$=x(x+5\sqrt{2})+\sqrt{2}(x+5\sqrt{2})=(x+5\sqrt{2})(x+\sqrt{2}).$$

**11.** (i) Here, $\frac{-2}{27}ab$ and $\frac{-20}{3}b$ are negative terms and $b$ occurs in both the terms.

So, we write $\frac{1}{9}b^2$ as $\left(\frac{-1}{3}b\right)^2$.

$$\therefore\ \frac{1}{81}a^2+\frac{1}{9}b^2+100-\frac{2}{27}ab-\frac{20}{3}b+\frac{20}{9}a$$

$$=\left(\frac{1}{9}a\right)^2+\left(\frac{-1}{3}b\right)^2+(10)^2+\left\{2\times\frac{1}{9}a\times\left(-\frac{1}{3}b\right)\right\}$$

$$+\left\{2\times\left(\frac{-1}{3}b\right)\times10\right\}+\left\{2\times10\times\frac{1}{9}a\right\}$$

**TR!CK**

$$a^2+b^2+c^2+2ab+2bc+2ac=(a+b+c)^2$$

$$=\left\{\frac{1}{9}a+\left(\frac{-1}{3}b\right)+10\right\}^2=\left(\frac{1}{9}a-\frac{1}{3}b+10\right)^2$$

(ii) Given that, $8(a+1)^2-2(a+1)(b+2)-15(b+2)^2$

Put $a+1=p$ and $b+2=q$, we have

$$8p^2-2pq-15q^2$$

Now,

$$8p^2-2pq-15q^2=8p^2-12pq+10pq-15q^2$$

$$=4p(2p-3q)+5q(2p-3q)$$

$$=(2p-3q)(4p+5q)$$

$$=\{2(a+1)-3(b+2)\}\{4(a+1)+5(b+2)\}$$

$$=\{2a+2-3b-6\}\{4a+4+5b+10\}$$

$$=\{2a-3b-4\}\{4a+5b+14\}.$$

# Chapter Test

## Multiple Choice Questions

**Q 1.** The number of zeroes in $x^2+6x+9$ is:

a. 1 b. 2

c. 3 d. None of these

**Q 2.** The value of $6.45\times6.45+12.9\times3.55+3.55\times3.55$ is:

a. 237 b. 126 c. 56 d. 100

## Assertion and Reason Type Questions

**Directions (Q. Nos. 3-4)** *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

**Q 3.** Assertion (A): The degree of the polynomial $(x+2)(x-3)(x+5)$ is 3.

Reason (R): The number of zeroes of a polynomial is the degree of that polynomial.

**Q 4.** Assertion (A): The zeroes of $p(x)=x^2+4x-5$ are –5 and 1.

Reason (R): A quadratic polynomial can have at most two zeroes.

## Fill in the Blanks

**Q 5.** Every real number is a .................. of the zero polynomial.

**Q 6.** The coefficient of $x^2$ in the polynomial $2x^3+4x^2+3x+1$ is ................ .

**True/False**

**Q 7.** The expression $\sqrt{t} + 5t - 1$ is a polynomial.

**Q 8.** The zeroes of the polynomial $p(x) = 2x^2 + 7x - 4$ are $-4$ and $\frac{1}{2}$.

**Case Study Based Questions**

**Q 9.** There is a bakery shop in our society. The chef bakes different varieties of cakes using different ingredients such as flour, chocolate, sugar etc. Based on the number of ingredients used in cakes, the price of individual cake is decided. The polynomial used to calculate price is $(x^2 - 3)100$, where $x$ denotes number of ingredients used in cake. Maximum ingredients used in cake are 12 and minimum is 5.

*On the basis of the above information, solve the following questions:*

(i) Find the price of cake having ingredient 5.

(ii) What are the zeroes of polynomial?

OR

What is the maximum price of cake?

(iii) Find the degree of polynomial.

**Very Short Answer Type Questions**

**Q 10.** Factorise $1 - 2ab - (a^2 + b^2)$.

**Q 11.** Factorise $x^2 + 5x - 66$.

**Short Answer Type-I Questions**

**Q 12.** If $x + \frac{1}{x} = 6$, then find the value of $x^3 + \frac{1}{x^3}$.

**Q 13.** Use factor theorem to show that $x^4 + 2x^3 - 2x^2 + 2x - 3$ is exactly divisible by $(x + 3)$.

**Short Answer Type-II Questions**

**Q 14.** If $x = (2+\sqrt{5})^{\frac{1}{2}} + (2-\sqrt{5})^{\frac{1}{2}}$ and $y = (2+\sqrt{5})^{\frac{1}{2}} - (2-\sqrt{5})^{\frac{1}{2}}$, find $x^2 + y^2$.

**Q 15.** (i) Factorise $25x^2 + 9y^2 + 9z^2 - 30xy - 18yz + 30xy$.

(ii) Evaluate $(97)^3$ by using suitable identity.

**Long Answer Type Question**

**Q 16.** Simplify:

$$\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$$